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# Hardness-temperature relationships in metals

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The high-temperature hardness data for thirty-seven metals have been collected from the published literature. Seven of these metals were further tested to confirm and improve the hardness-temperature data. All data were analysed to calculate the softening parameter $B$ and the apparent activation energy for indentation $B'$. The values of $B'$ are compared with the activation energies for lattice and dislocation pipe diffusion to obtain an estimate of the stress coefficient, $\alpha$, for creep. The $B'$ and $\alpha$ values have been improved by considering the effect of elastic modulus.

## 1. Introduction

The variation of hardness with temperature is similar to the variation of a number of mechanical properties such as flow stress [1]. The hardness-temperature relationship was first suggested by Ito [2] and Shishokin [3], and has been explored subsequently by a number of investigators [4-8]. The relation is given as

$$H = A \exp(-BT) \quad (1)$$

where $H$ is hardness equivalent to the mean compressive stress and $T$ temperature in K. The constant, $A$, is extrapolated "intrinsic hardness" i.e. hardness at $T = 0$ and constant $B$ is the softening coefficient of hardness.

The constants $A$ and $B$ have one set of values ($A_1$, $B_1$) at low temperatures and another set ($A_2$, $B_2$) at higher temperatures, suggesting a change of mechanism. The transition between the low- and high-temperature behaviours may occur at one temperature or over a range of temperatures. In most metals and alloys, the transition temperature $T_t$ is about half the melting temperature ($T_m$).

Another way of treating the hardness data above $T_t$ is based on Arrhenious equation relating the strain rate $\dot{\epsilon}$ during indentation to temperature:

$$\dot{\epsilon} = A' \exp(-B'/RT) \quad (2)$$

where $R$ is universal gas constant, $B'$ apparent "activation energy" during indentation and the true strain during spherical indentation is given as [9]

$$\epsilon = 0.2\, d/D\,. \quad (3)$$

Here $d$ is diameter of indentation and $D$ diameter of indenter. If the short-time indentation process can be expressed by the time law of the type

$$d = d_0 + k(t/t_0)^n \quad (4)$$

where $d_0$ is initial indentation at time $t_0$, $k$ and $n$ constants, it is clear that $\dot{d} \sim d$ for constant $n$ and indentation time. Combining Equations 2, 3 and 4 we have

$$d = A'' \exp(-B'/RT) \quad (5)$$

where $A''$ is a constant.

The stresses acting on the surface during indentation are essentially compressive and their mean value is given by [9]

$$p = \frac{4W}{\pi d^2} \quad (6)$$

where $W$ is load. It is found that $p$ is very high initially, but gradually decreases as the size of the indentation increases with time. Ordinarily the indentation size attains a fairly constant value some time after the start of indentation, and this value corresponds to hardness. In terms of the projected area of indentation, Equation 6 represents hardness; however, in terms of true surface area, the hardness $H$ is given as

$$H = 2W/\pi d[D - \sqrt{[D^2 - d^2]} \quad (7)$$

If $d/D$ is small, or more specifically $(d/D)^2 \ll 1$, $H \sim d^{-2}$ in Equation 7. Substituting $H \sim d^{-2}$ in Equation 5 for $R = 2$ cal (g. mol)$^{-1}$,

$$H = A''' \exp(B'/T) \quad (8)$$

where $A'''$ is a constant and $B'$ retains the units of energy. Thus above $T_t$, the plot of ln $H$ against $1/T$ should yield a straight line with its slope representing the apparent activation energy for indentation.

We elaborate the derivation of Equation 8 to emphasize the underlying assumptions: $\epsilon \sim d$, $\dot{d} \sim d$ and $(d/D)^2 \ll 1$. For non-spherical indenters, Equation 3 is not valid and for soft metals $(d/D)^2$ is large when high loads are used. Under the circumstances, Equation 8 becomes essentially empirical. We further note that although the hardness data can be fitted on both Equations 1 and 8, the equations are basically incompatible. If data were accurate and numerous enough to obtain the exact fit on Equation 1, the ln $H$ against $1/T$ plot would be a curve instead of a straight line, i.e. $B'$ would change gradually with temperature from low to high value.

At temperatures above $T_t$, the indentation size does not attain a constant value after a certain time but continues to increase with time [10]. The creep under indentation may be given by the Equation [11]

$$\dot{\epsilon} = K p^{\alpha} \exp(-Q_c/RT) \qquad (9)$$

where $\alpha$ is stress coefficient, $Q_c$ activation energy for creep and $K$ a constant. Equations 8 and 9 are related as [12, 13]

$$Q_c = 2\,\alpha\, B' . \qquad (10)$$

Sherby [13] has shown that for the short-time hardness testing, $Q_c$ is identical with the activation energy for lattice diffusion ($Q_L$) and the power-law is valid above about 0.75 $T_m$, while $Q_c$ equals the activation energy for dislocation pipe diffusion ($Q_d \approx \frac{2}{3} Q_L$) [14] and the power-law breaks down near about 0.6 $T_m$.

In what follows we collect the hardness versus temperature data for thirty-seven metals from various sources [8, 13, 15-29] and analyse the data in view of Equations 1, 8 and 10. The various hardness-temperature parameters are calculated and an estimate of the stress coefficient $\alpha$ is made. The Brinell hardness of seven metals Bi, Sn, Cd, Pb,Zn, Sb and Al are measured from 0.5 to 0.9 $T_m$, using the procedure outlined elsewhere [12], to provide the additional data for comparison.

## 2. Analysis and discussion

The basic contradiction between Equations 1 and 8 is illustrated in Fig. 1 for gold [21]. The left-hand graph ln $H$ against $1/T$, is plotted with the actual data points whereas the right-hand graph is plotted with the line points from the ln $H$ against $T$ correlation. Similar plot for the modulus corrected hardness [13] $(H/E)$ is also shown in the figure using the elastic modulus ($E$) given by Lozinskii [21].

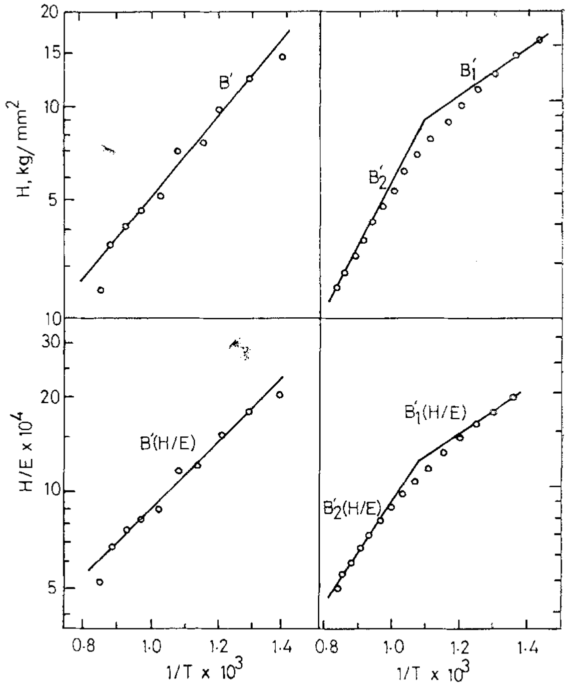


*Figure 1* Temperature dependency of Vickers hardness and modulus corrected hardness of Gold.

Although a unique line through the data points in the left-hand plots seems reasonable, more accurate data fitting Equation 1 with little scatter would have revealed gradually increasing $B'$ and $B'$ $(H/E)$ with temperature as shown in the right-hand plots. As an approximation the varying $B'$ can be represented by two slopes $B_1'$ and $B_2'$, the transition occurring over a range of temperatures and representing a change in mechanism from the one controlled by the dislocation pipe diffusion at lower temperatures to the one controlled by lattice diffusion at higher temperatures, as pointed out by Sherby and Armstrong [13].

Table I lists the sources for the available hardness data, representing a variety of testing methods: Vickers, cone, Brinell. Also listed in the Table are the calculated values of the softening parameters $B_1$, $B_2$ and the activation energies $B'$, $B_2'$ $(H/E)$. In calculating these values, judicious choice of consistent hardness data has been made. The $B_2'$ $(H/E)$ has been calculated from the data points above 0.75 $T_m$ rather than

TABLE I *Hardness-temperature data for metals**

| Metal | $T_m$(K) | $(T/T_m)$ range | $B_1 \times 10^4$ | $B_2 \times 10^4$ | $B'$ | $B_2'(H/E)$ | $Q_L$† | References‡ |
|---|---|---|---|---|---|---|---|---|
| In | 429 | 0.68–0.90 | — | 90.0 | 0.92 | 1.35 | 16.0 | 8 (V); 15 (C) |
| Sn | 505 | 0.02–0.95 | 50.0 | 94.9 | 1.20 | 1.25 | 18.0 | 8 (V); 15 (C) |
| Bi | 544 | 0.15–0.85 | 32.4 | 76.4 | 1.10 | 1.50 | 18.0 | 8 (V); 16 (B) |
| Tl | 575 | 0.38–0.97 | 40.0 | 102.5 | 1.50 | 2.00 | 22.8 | 8 (V); 15, 17 (B) |
| Cd | 594 | 0.10–0.89 | 40.0 | 108.0 | 1.49 | 2.07 | 22.0 | 8, 18 (V); 15, 16 (B) |
| Pb | 600 | 0.13–0.87 | 29.0 | 70.0 | 1.94 | 3.00 | 27.9 | 8, 18 (V) |
| Zn | 693 | 0.02–0.98 | 45.2 | 97.9 | 1.80 | 3.06 | 25.0 | 8, 18 (V); 15, 16, 19, 20 (B) |
| Sb | 903 | 0.09–0.84 | 20.2 | 40.7 | 2.00 | 2.30 | 39.0 | 8 (V), 16 (B) |
| Mg | 924 | 0.32–0.93 | 28.0 | 76.0 | 3.03 | 3.60 | 32.0 | 18 (V), 20 (B) |
| Al | 931 | 0.08–0.94 | 23.3 | 72.0 | 2.40 | 3.20 | 34.0 | 8, 18, 21 (V); 16 (B) |
| Ba | 998 | 0.29–0.87 | 25.0 | 52.0 | 1.29 | 2.52 | 16.0 | 15 (C) |
| Nd | 1019 | 0.06–0.23 | 23.9 | — | — | — | — | 21 (V) |
| Sr | 1030 | 0.28–0.95 | — | 41.8 | 1.29 | — | — | 15 (C) |
| Ce | 1077 | 0.20–0.77 | 6.3 | 66.0 | 2.88 | — | — | 15 (C) |
| La | 1099 | 0.07–0.96 | 11.0 | 61.0 | 4.05 | 4.42 | 38.0 | 15 (C) |
| Ca | 1118 | 0.26–0.96 | 9.3 | 78.0 | 5.33 | 5.97 | 80.5 | 15 (C) |
| Ge | 1210 | 0.70–0.99 | 10.9 | 44.0 | 7.20 | 7.20 | 75.0 | 15 (C) |
| Pr | 1213 | 0.25–0.90 | 9.9 | 38.2 | 2.02 | — | — | 15 (C) |
| Ag | 1234 | 0.24–0.95 | 9.3 | 35.4 | 2.78 | 3.61 | 43.5 | 21 (V) |
| Au | 1336 | 0.20–0.88 | 10.0 | 38.0 | 2.86 | 5.02 | 45.7 | 21 (V) |
| Cu | 1356 | 0.05–0.90 | 12.3 | 39.0 | 3.04 | 4.44 | 48.2 | 8, 18, 21, 22 (V); 15 (C) |
| α-U | 1410 | 0.65–0.84 | — | 30.4 | 2.98 | — | 46.0 | 8, 17, 23 (V) |
| Be | 1551 | 0.19–0.82 | 15.0 | 44.0 | 3.75 | 4.10 | 38.4 | 20, 21 (V); 20 (B) |
| Si | 1683 | 0.22–0.64 | 9.5 | 52.5 | 2.87 | — | — | 21 (V, B) |
| Ni | 1728 | 0.01–0.85 | 10.0 | 39.0 | 5.07 | 5.50 | 69.5 | 8, 16, 18, 21 (V) |
| Co | 1768 | 0.13–0.75 | 5.8 | 26.0 | 3.93 | — | 61.9 | 17, 21 (V); 15 (C) |
| α-Fe | 1808 | 0.01–0.66 | 3.3 | 50.0 | 4.58 | — | 67.0 | 6, 8, 17, 21 (V) |
| γ-Fe | 1808 | 0.66–0.85 | — | — | 5.20 | — | 68.0 | |
| Pd | 1822 | 0.16–0.80 | 5.1 | 32.2 | 3.12 | — | 63.6 | 20, 21 (V) |
| Pt | 2046 | 0.42–0.66 | 11.0 | 37.3 | 5.47 | — | 68.2 | 20, 21 (V) |
| α-Ti | 2073 | 0.14–0.56 | 10.2 | — | — | — | 44.0 | 8, 17, 18, 20, 21 (V); 20 (B) |
| β-Ti | 2073 | 0.56–0.80 | — | 68.2 | 5.30 | — | | |
| Cr | 2163 | 0.04–0.68 | 5.5 | 23.0 | 3.89 | — | 73.2 | 15 (C) |
| Rh | 2239 | 0.13–0.60 | 12.1 | 22.8 | 4.34 | — | 87.0 | 21, 24 (V) |
| Ir | 2727 | 0.10–0.48 | 7.4 | 42.1 | 5.21 | — | 100.0 | 20, 21 (V) |
| Nb | 2742 | 0.11–0.79 | 12.0 | 37.6 | 5.61 | — | 96.0 | 20, 25 (V) |
| Mo | 2883 | 0.10–0.78 | 7.3 | 29.1 | 6.36 | — | 96.9 | 8, 21, 26, 27 (V) |
| Ta | 3269 | 0.09-0.64 | 7.5 | 26.6 | 8.45 | — | 98.7 | 27, 28 (V) |
| W | 3683 | 0.04–0.57 | 8.2 | 20.3 | 10.00 | — | 140.0 | 8, 26, 27, 29 (V) |

*$B_1$,$B_2$ are in $K^{-1}$ and $B'$, $B'_2(H/E)$, $Q_L$ in kcal (g at)$^{-1}$
†Lattice self-diffusion data from references 20, 30, 31
‡Testing method: V = Vickers, C = Cone, B = Brinell.

the line points as in Fig. 1. In so far as the slope change between 0.75 and 0.95 $T/T_m$ is rather small, the use of the data points rather than line points would yield reasonable values of $B_2'$ $(H/E)$. Occasionally, insufficient data or phase transformations did not permit calculation of some of the quantities.

Fig. 2 relates $B_1$, $B_2$, $B'$ and $B_2'$ $(H/E)$ to melting temperature. The general trend is to decrease the softening parameters and increase the activation energies with $T_m$. The $B_2'$ $(H/E)$ for the high melting ($T_m > 1750$K) metals has not been calculated due to non-availability of the hardness data above $T/T_m$ of about 0.75. Fig. 3 relates $B'$ and $B_2'$ $(H/E)$ to $Q_c$, the value of $Q_c$ depending upon the temperature range considered [13, 14]. In this correlation the following $Q_c$ values were used: for $T/T_m$ = 0.5 to 0.7,

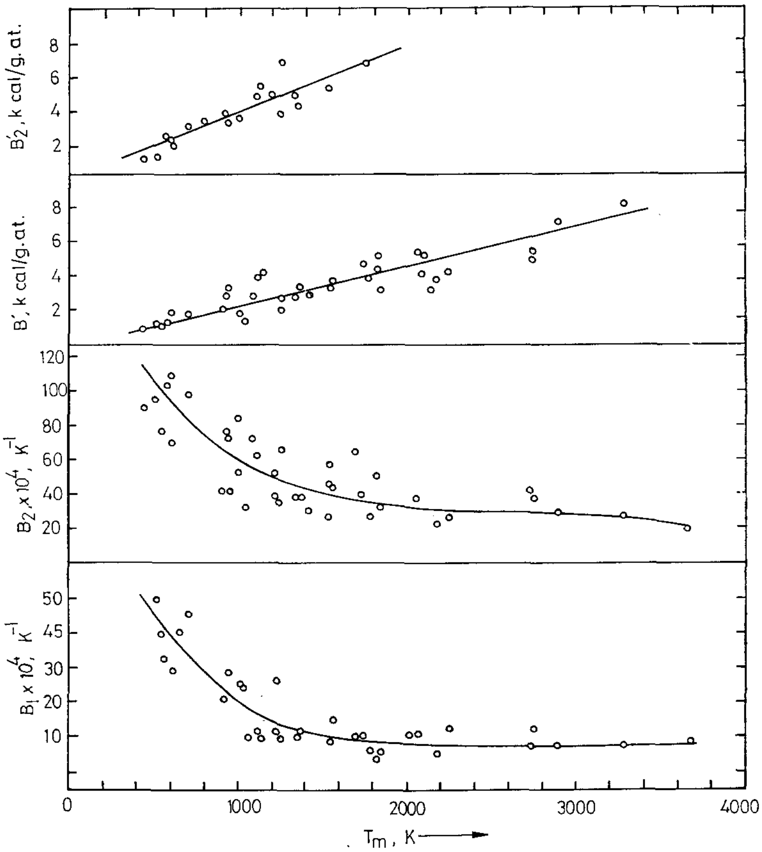


*Figure 2* Relationship between $B_1$, $B_2$, $B'$, and $B_2'$ and melting temperature.

$Q_c \approx Q_d \approx (2/3)Q_L$; for $T/T_m = 0.7$ to 0.95, $Q_c \approx Q_L$; and for $T/T_m = 0.5$ to 0.95, $Q_c \approx (1/2)(Q_d + Q_L) \approx (5/6)Q_L$. The values of the lattice self-diffusion obtained from literature [20, 30, 31] are also listed in Table I. There is a considerable deviation from $\alpha = 5$ characteristic of the "five" power law, even in the case of data above 0.75 $T_m$, where power-law relation is expected to be valid.

The temperature dependence of the elastic modulus is similar to that of the hardness. Frequently a break is observed in the ln $E$ against $T$ plot at about 0.5 $T_m$, yielding two slopes $B_1$ ($E$) and $B_2$ ($E$) below and above the transition temperature. The $B_1$ ($E$) and $B_2$ ($E$) values were calculated using the data from Lozinskii [21] and are correlated with the hardness softening parameters $B_1$ and $B_2$ respectively. The low melting metals which soften rapidly also decrease their elastic modulus rapidly with temperature, as shown in Fig. 4.

The scatter in Figs. 2 to 4 may be due to several reasons: variety of indenters and loads, different indentation times, sketchiness of the available data, and the unknown purity and grain size of the metals. Typical examples of the variation of $B'$ are shown in Fig. 5 for tin and

TABLE II Variation in $B'$ and $B_2'(H/E)$

| Metal | $B'$, kcal (g at)$^{-1}$ | | | $B_2'(H/E)$, kcal (g at)$^{-1}$ | |
|---|---|---|---|---|---|
| | This study | Larsen-Badse | Table I | This study | Table I |
| Sn | 0.73 | 1.10 | 1.20 | 1.00 | 1.25 |
| Bi | 1.07 | 1.27 | 1.10 | 1.43 | 1.50 |
| Cd | 1.62 | 1.72 | 1.49 | 2.01 | 2.07 |
| Pb | 1.75 | 2.02 | 1.94 | 2.80 | 3.00 |
| Zn | 2.08 | 1.89 | 1.80 | 3.00 | 3.06 |
| Sb | 1.78 | 2.01 | 2.00 | 2.10 | 2.30 |
| Al | 2.18 | 2.84 | 2.40 | 3.10 | 3.20 |

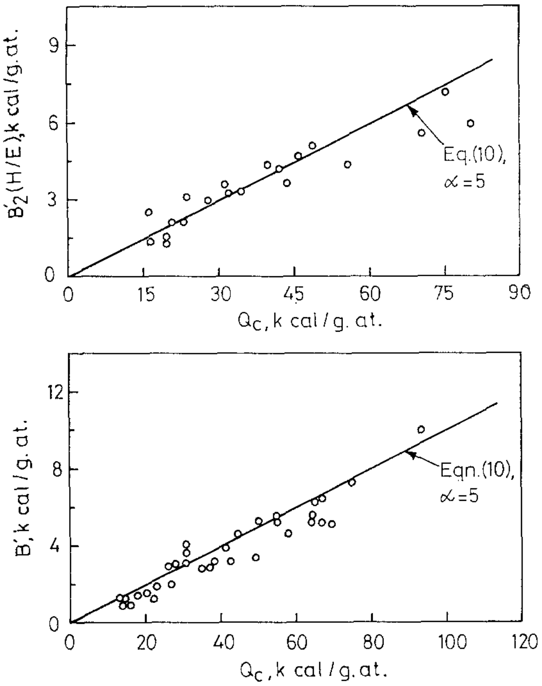


*Figure 3* Relation between activation energies for indentation and for creep.

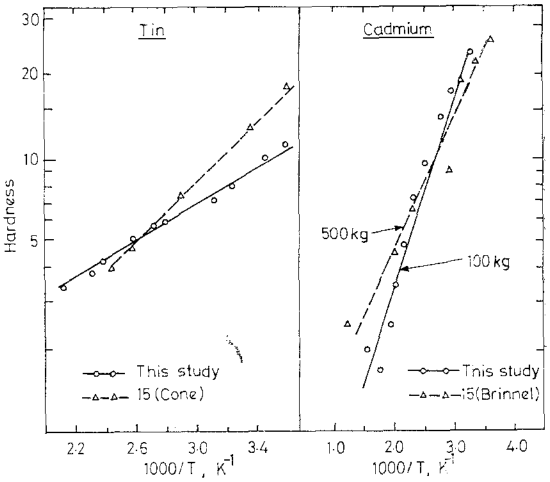


*Figure 5* Temperature dependence of hardness for Tin and Cadmium.

cadmium. The Brinell hardness data obtained in this study with 10 mm diameter indenter and 100 kg load are compared with the cone hardness data for tin [15] and with the Brinell data using 500 kg load for cadmium [15]. The $B'$ and $B_2'$ $(H/E)$ values obtained in this study for the seven metals are compared with those available in the literature and are listed in Table II.

Several facts emerge from the results: the $B$ and $B'$ values for a given metal are not necessarily unique, their precise determination is difficult and subject to the variables mentioned earlier. Furthermore, drawing a straight line through the data points itself is subject to certain variation. The $\alpha$-values calculated from Equation 10 can at best be only approximate. It is hazardous to make more specific deductions about the deformation behaviour beyond the simple correlations unless more accurate and standardized data become available.

## 3. Conclusions

The softening parameter of hardness ($B$) is inversely related and the apparent activation energy for indentation ($B'$) is directly related to the melting temperature $T_m$ of the metal. The

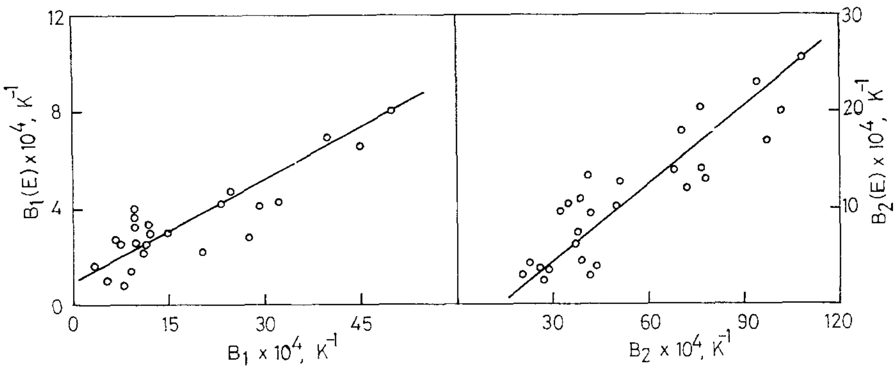


*Figure 4* Relationship between softening parameters of elastic modulus and hardness.

softening parameter shows abrupt change below ($B_1$) and above ($B_2$) about 0.5 $T_m$. The elastic modulus softening parameters $B_1$ ($E$) and ($B_2$ $E$) are readily corrected with the hardness parameters $B_1$ and $B_2$. The unique value of $B'$ is difficult to define, partly due to its variation with temperature and partly due to its dependence on testing and metal conditions. The value of $B'$ at low temperature ($< 0.75$ $T_m$, $B_1'$) may be correlated with the activation energy for the dislocation pipe diffusion and that at the high temperature ($> 0.75$ $T_m$, $B_2'$) with the activation energy for lattice self-diffusion. The correlation yields an approximate value of the stress coefficient ($\alpha$) which is close to 5.

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